Pythagorean Theorem

Lesson Synopsis:

In this lesson, students model the Pythagorean Theorem concretely using area models while making connections to algebra. In addition, students explore Pythagorean triples by constructing triangles and explore the relationships between dilations, similar triangles, and the Pythagorean Theorem. Students apply the Pythagorean Theorem and Pythagorean triples in real-world contexts to solve problems.

TEKS:

G.2	***Geometric structure. The student analyzes geometric relationships in order to make and verify conjectures.***
G.2B	**Make conjectures about angles, lines, polygons, ~~circles, and three-dimensional figures~~ and determine the validity of the conjectures, choosing from a variety of approaches such as coordinate, transformational, or axiomatic.**
G.5	***Geometric patterns. The student uses a variety of representations to describe geometric relationships and solve problems.***
G.5B	**Use numeric and geometric patterns to make generalizations about geometric properties, including properties of polygons, ratios in similar figures ~~and solids,~~ and angle relationships in polygons ~~and circles.~~**
G.5D	**Identify and apply patterns from right triangles to solve meaningful problems, including ~~special right triangles (45-45-90 and 30-60-90) and~~ triangles whose sides are Pythagorean triples.**
G.8	***Congruence and the geometry of size. The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations.***
G.8C	**Derive, extend, and use the Pythagorean Theorem.**
G.11	***Similarity and the geometry of shape. The student applies the concepts of similarity to justify properties of figures and solve problems.***
G.11C	**Develop, apply, and justify triangle similarity relationships, such as right triangle ratios, ~~trigonometric ratios,~~ and Pythagorean triples using a variety of methods.**

GETTING READY FOR INSTRUCTION

Performance Indicator(s):

- Analyze patterns to derive and make conjectures about the Pythagorean Theorem and Pythagorean Triples. Extend both to identify right triangles and find missing side lengths in right triangles. (G.2B; G.5A, G.5B, G.5D; G.8C; G.11C)
  ELPS ELPS: 1E, 1H, 2E, 2I, 3H, 4F, 5G

Key Understandings and Guiding Questions:

- The Pythagorean Theorem, its converse, and Pythagorean Triples can be used to identify right triangles and calculate the measurement of their sides.
  – How is similarity used to generate Pythagorean triples?
  – How can the Converse of the Pythagorean Theorem be used to classify triangles by angles?
  – How can Pythagorean triples be used to solve right triangles?

Vocabulary of Instruction:

- Pythagorean Theorem
- right triangles
- leg of right triangle
- hypotenuse
- Pythagorean triples

Materials:

- transparency marker
- patty paper
- colored pencils
- scissors
- grid paper

Resources:

- **State Resources**
  — **Mathematics TEKS Toolkit:** Clarifying Activity/Lesson,/Assessments
  http://www.utdanacenter.org/mathtoolkit/index.php

  — **TEXTEAMS: Geometry for All Institute:** I – Functionally Speaking with Reason; Act. 4 (Perplexing Puzzles), Act. 5 (Premeditated Patty Paper Proof), Act. 6 (Patty Paper Proof – The Sequel), Act. 7 (Pythagoras and the President), Act. 8 (Tantalizing Triples), Act. 9 ("Pythagoras:" A Triples Tune)

  — **TEXTEAMS: High School Geometry: Supporting TEKS and TAKS:** III – Triangles; 3.0 The Pythagorean Theorem, 3.1, Act. 1 (Fishing Rod); 4.0 Student Activity – The Pythagorean Theorem, 4.1, Act. 1 (Patty Paper Proof), 4.2, Act. 2 (Pythagorean Theorem – More or Less), 4.3, Act. 3 (Distance in Space)

Advance Preparation:

1. Handout: **Super Bowl Party** (1 per group)
2. Transparency: **Super Bowl Party** (1 per teacher)
3. Handout: **Modeling the Pythagorean Theorem** (1 per student)
4. Handout: **Pythagorean Theorem Practice** (1 per student)
5. Handout: **Discovering Pythagorean Triples** (1 per student)
6. Handout: **Applications of Pythagorean Theorem and Pythagorean Triples** (1 per student)
7. Handout: **Super Bowl Party Revisited** (1 per student)

Background Information:

This lesson builds on previous content from the lessons about dilations and similarity. The student explorations require students to have a general working knowledge of dilations, similar figures and scale factor. Since students model the Pythagorean Theorem with area models, students are required to find the area of triangles, rectangles and squares.

GETTING READY FOR INSTRUCTION SUPPLEMENTAL PLANNING DOCUMENT

Instructors are encouraged to supplement, and substitute resources, materials, and activities to differentiate instruction to address the needs of learners. The Exemplar Lessons are one approach to teaching and reaching the Performance Indicators and Specificity in the Instructional Focus Document for this unit. A Microsoft Word template for this planning document is located at www.cscope.us/sup_plan_temp.doc. If a supplement is created electronically, users are encouraged to upload the document to their Lesson Plans as a Lesson Plan Resource in your district Curriculum Developer site for future reference.

INSTRUCTIONAL PROCEDURES

Instructional Procedures

ENGAGE

1. Put students in groups of 3-4.
2. Distribute the handout: **Super Bowl Party** to each group.
3. Have one student in the group read the scenario of **Super Bowl Party**. Have members of the group discuss the situation and determine information that would be needed to solve the problem. Have them write items on the handout.
4. Display the transparency: **Super Bowl Party** and have each group record on the transparency one of their items of information they have listed. Once all groups have responded, ask if any groups have any to add to the list. Continue the class discussion with the questions below.
   Facilitation Questions:
   - **What additional information is needed before you can select the best TV for your home theater room?** *The dimensions of the opening in the cabinet are needed.*
   - **What is meant by "largest" TV? How could this be measured?** *This could mean the largest diagonal measure because that is the way TVs are sized. It could mean the largest screen area.*

Notes for Teacher

NOTE: 1 Day = 50 minutes
Suggested Day 1 (1/2 day)

MATERIALS
- Handout: **Super Bowl Party** (1 per group)
- Transparency: **Super Bowl Party** (1 per teacher)
- transparency marker

TEACHER NOTE
The purpose of this activity is to pose a problem that can be solved using the Pythagorean Theorem.

TEACHER NOTE
Students will address this problem again in the assessment portion of the lesson and arrive at a solution. At this point, students do not have enough

Instructional Procedures

- **What is meant by an aspect ratio? What does 4:3 mean? What does 16:9 mean?** *The ratio compares the width to the height. A 4:3 ratio means for every 4 units of width there are 3 units of height; similarly, for every 16 units of width there are 9 units of height.*
- **Which aspect ratio has the greater area for a given diagonal size?** *The 4:3 ratio will yield more area than the 16:9 for a given diagonal length. (This may not be obvious to students without a lot of thought. This particular question is revisited in the assessment portion of the lesson.)*
- **What topic from mathematics shows the relationship between the width, height, and diagonal length of a rectangle?** *The Pythagorean Theorem.*

EXPLORE 1

1. Have students continue to work in their small groups.
2. Distribute the handout: **Modeling the Pythagorean Theorem** to each student. Make sure each group has the necessary materials to complete the activity (patty paper, scissors, colored pencils).
3. Have the group work together to complete Area Model I on pp. 1-2.
4. Have groups share out results in whole-group discussion.
5. Have the group work together to complete Area Model II on pp. 3-4.
6. Have groups share out results in whole-group discussion.
7. Have the group work together to complete An Algebraic Approach on pp. 5-6.
8. Have groups share out results in whole-group discussion.
9. If necessary, students can complete the questions for An Algebraic Approach on pp. 5-6 for homework.

Notes for Teacher

information to solve or even formulate a solution to the problem. Use student responses to lead a discussion about what additional information is needed to solve this problem. Some questions to help facilitate the discussion are below.

STATE RESOURCES
Mathematics TEKS Toolkit: Clarifying Activity/Lesson/Assessment may be used to reinforce these concepts or used as alternate activities.

Suggested Days 1-2 (1 ½ days)
MATERIALS

- Handout: **Modeling the Pythagorean Theorem** (1 per student)
- patty paper
- scissors
- colored pencils

TEACHER NOTE
The purpose of this activity is to give students the opportunity to model the Pythagorean Theorem concretely and connect to the numerical relationship $a^2 + b^2 = c^2$. Students model the Pythagorean Theorem concretely with two different area models by showing that the square formed by the length of the hypotenuse of a right triangle has area equal to the sum of the areas of the squares formed by the lengths of the legs. Students then connect an area model with the numerical representation $a^2 + b^2 = c^2$.

TEACHER NOTE
Transparency cutouts of each of the models can be made before class and used on the overhead, if students are having trouble seeing how to arrange the pieces.

STATE RESOURCES
TEXTEAMS: Geometry for All Institute: I – Functionally Speaking with Reason; Act. 4 (Perplexing Puzzles), Act. 5 (Premeditated Patty Paper Proof), Act. 6 (Patty Paper Proof – The Sequel), Act. 7 (Pythagoras and the President) may be used to reinforce these concepts or used as alternate activities.

Instructional Procedures

Notes for Teacher

TEXTEAMS: High School Geometry: Supporting TEKS and TAKS: III – Triangles; 4.0 Student Activity – The Pythagorean Theorem, 4.1, Act. 1 (Patty Paper Proof) may be used to reinforce these concepts or used as alternate activities.

EXPLAIN 1

1. Debrief the previous activity handout: **Modeling the Pythagorean Theorem** by facilitating a discussion of how the area models illustrate the Pythagorean Theorem. Some questions to facilitate discussions are below.
   Facilitation Questions:
   - **In the area models, what do the squares with side length of each leg represent numerically?** *The square of the length of a leg; from the Pythagorean Theorem,* a^2 *and* b^2*.*
   - **In the area models, what does the square with side length of the hypotenuse represent numerically?** *The square of the length of the hypotenuse; from the Pythagorean Theorem,* c^2*.*
   - **How does each area model verify the Pythagorean Theorem?** *The figures that make up the squares of each leg can be rearranged to exactly fill the square of the hypotenuse.*
   - **How does the Algebraic Approach verify the Pythagorean Theorem?** *Since the two squares have the same dimensions, they are congruent. Since they are congruent, they have equal areas. The area of Figure 1 is* $a^2 + 2ab + b^2$*; the area of Figure 2 is* $2ab + c^2$*. Equating these two expressions yields* $a^2 + 2ab + b^2 = 2ab + c^2$*. Simplifying yields* $a^2 + b^2 = c^2$*.*
2. Distribute the handout: **Pythagorean Theorem Practice**
3. Go over p. 1 to formally introduce the Pythagorean Theorem, its converse, and related corollaries. Model one of the problems from 1-10 and one of the problems from 11-15 of handout: **Pythagorean Theorem Practice** to illustrate the theorems.
4. Have students work individually on the remaining problems from handout: **Pythagorean Theorem Practice**. This may be completed for homework, if necessary.

Suggested Day 3

MATERIALS

- Handout: **Pythagorean Theorem Practice** (1 per student)

TEACHER NOTE

The third model of the previous EXPLORE phase attempts to connect an area model to the numerical representation $a^2 + b^2 = c^2$. Although question 5 instructs students to find the area of Figure 1 by summing the individual areas of the figure, do not miss the opportunity to point out that the area could also be found by squaring the binomial $(a + b)$ or $(a + b)^2$ which is $(a + b)(a + b)$. This is a powerful connection to algebra that some students may miss.

TEACHER NOTE

Due to the availability of technology and the fact that students are able to use calculators on the TAKS test, techniques used to simplify radicals are being emphasized less. For example, $\sqrt{48}$ has traditionally been reduced to $4\sqrt{3}$. While some students may be able to do this and there is merit in being able to simplify radicals, be careful that this does not become the focus of the lesson as students solve right triangle problems that result in radicals. It is more important that students understand how to use technology to reconcile their solution than it is to simplify to reduced form. Although this is not the traditional viewpoint, it does not limit a student's ability to perform on TAKS.

TEACHER NOTE

Be careful to remind students that while the **Converse of the Pythagorean Theorem** is presented as a theorem and can be proven, not all converses of conditionals are true as seen in the unit

Instructional Procedures

Notes for Teacher

over conditionals and their related statements.

STATE RESOURCES
TEXTEAMS: High School Geometry: Supporting TEKS and TAKS: III – Triangles; 3.0 The Pythagorean Theorem, 3.1, Act. 1 (Fishing Rod); 4.0 Student Activity – The Pythagorean Theorem, 4.2, Act. 2 (Pythagorean Theorem – More or Less), 4.3, Act. 3 (Distance in Space) may be used to reinforce these concepts or used as alternate activities.

EXPLORE/EXPLAIN 2

1. Distribute copies of handout: **Discovering Pythagorean Triples** to students.
2. Go over the instructions for the activity on p. 1 and have students work in pairs or small groups to complete #1-4 on the first page. Make sure each group has grid paper to make rulers and also draw the right triangles.
3. Debrief answers to question #4 by having each group share one of their findings and record them on the board or overhead. When each group has shared, ask if any other observations were made.
4. Have students continue working with a partner or small group to complete pp. 2-3.
5. If students do not finish the activity in class, it may be completed as homework.

Suggested Day 4
MATERIALS
- Handout: **Discovering Pythagorean Triples** (1 per student)
- grid paper

TEACHER NOTE
The purpose of this activity is to help students discover common Pythagorean triples that can be used to solve right triangles. Students investigate Pythagorean triples by constructing triangles with integer leg lengths using grid paper and verify which triangles result in an integer hypotenuse length. Students also investigate Pythagorean triples while connecting to previous knowledge of dilations, similar figures, and scale factor.

STATE RESOURCES
TEXTEAMS: Geometry for All Institute: I – Functionally Speaking with Reason; Act. 8 (Tantalizing Triples), Act. 9 ("Pythagoras:" A Triples Tune) may be used to reinforce these concepts or used as alternate activities.

ELABORATE

1. Debrief the handout: **Discovering Pythagorean Triples** by facilitating a discussion of Pythagorean triples and their applications. Some questions to help guide the discussion are below.
   Facilitation Questions:
   - **What are some of the Pythagorean triples you discovered in the grid paper activity?** *Unique Pythagorean triples are as follows: 3, 4, 5; 5, 12, 13; 7, 24, 25; 8, 15, 17; and others. (Students may not have all unique triples, i.e. students may have 3, 4, 5 as well as 6, 8, 10,*

Suggested Day 5
MATERIALS
- Handout: **Applications of Pythagorean Theorem and Pythagorean Triples** (1 per student)

TEACHER NOTE
The purpose of this activity is to give

Instructional Procedures

etc.)

- **How did you verify that the triples represented right triangles?** *Concretely, the students observed that the integer triples resulted in right triangles by measuring the hypotenuse with their grid paper rulers. From the table, the sum of the squares of the two shorter sides equaled the square of the longest side. (Converse of the Pythagorean Theorem)*
- **What happened when a triangle represented by a Pythagorean triple was dilated by a scale factor of 2? …scale factor of 3?** *The dilation by a scale factor of 2 resulted in a new Pythagorean triple whose integers were two times the original. The dilation by a scale factor of 3 resulted in a new Pythagorean triple whose integers were 3 times the original.*
- **Based on your exploration of Pythagorean triples using dilations and similar triangles, describe a procedure for generating new Pythagorean triples given a beginning triple?** *Multiply the original triple by a positive integer.*
- **Do you think multiplying a Pythagorean triple by any number would result in a set of numbers that represent right triangle side lengths?** *No. 3, 4, 5 scaled by -1 yields -3, -4, -5 which satisfies the relationship $a^2 + b^2 = c^2$; however, there are no triangles with negative side lengths. If we qualify the scalar to be positive, we can safely answer yes.*
- **How can Pythagorean triples be used to solve right triangles?** *By recognizing two of the three terms of a triple (or multiples, thereof) the third term can be recalled from memory.*

1. Distribute the handout: **Applications of Pythagorean Theorem and Pythagorean Triples** to each student.
2. Have students complete **Applications of Pythagorean Theorem and Pythagorean Triples**. This may be completed in pairs or independently. Monitor students to check for understanding.

Notes for Teacher

students an opportunity to generate Pythagorean triples and practice the Pythagorean theorem in real-world contexts.

EVALUATE

1. Debrief handout: **Applications of Pythagorean Theorem and Pythagorean Triples** in whole-group discussion as a review before the assessment.
2. Distribute the handout: **Super Bowl Party Revisited** to each student.
3. Have students complete **Super Bowl Party Revisited** individually as an assessment of student conceptual understanding.

Suggested Day 6

MATERIALS

- Handout: **Super Bowl Party Revisited** (1 per student)

TEACHER NOTE

This activity should be completed independently to assess student knowledge of the concepts taught in the lesson.

TEACHER NOTE

In **Super Bowl Party Revisited** students use their knowledge of the Pythagorean Theorem and Pythagorean triples to solve the dilemma involving TVs with different aspect ratios.

TAKS CONNECTION

Grade 9 TAKS 2003 #14,38
Grade 10 TAKS 2003 #11,13,56
Grade 11 TAKS 2003 #20,41

Instructional Procedures	Notes for Teacher
	Grade 9 TAKS 2004 #26,33 Grade 10 TAKS 2004 #15,36 Grade 11 TAKS 2004 #25,46,51 Grade 11 July TAKS 2004 #21,25,41 Grade 9 TAKS 2006 #23,35 Grade 10 TAKS 2006 #15,53 Grade 11 TAKS 2006 #2 Grade 11 July TAKS 2006 #10,29

Super Bowl Party KEY

Congratulations! You bragged a little too much about the home theater room that you are adding to your home and your friends have talked you into having a Super Bowl party in a few weeks. The problem is you are missing the all important TV set! You have been shopping for TV sets and have only become confused. You want the largest TV set that will fit the cabinet that you just had custom built for your home theater room. Here is what you have learned.

- TV screens are sized according to the measure of the diagonal of the rectangular screen.
- There are different aspect ratios for different types of TV's.
    - Traditional TV's have an aspect ratio of 4:3 (width: height).
    - High definition TV sets (HDTV) have an aspect ratio of 16:9 (width: height).

What additional information is needed before you can select the best TV for your home theater room?

Facilitation Questions:

- **What additional information is needed before you can select the best TV for your home theater room?** *The dimensions of the opening in the cabinet are needed.*
- **What is meant by "largest" TV? How could this be measured?** *This could mean the largest diagonal measure because that is the way TVs are sized. It could mean the largest screen area.*
- **What is meant by an aspect ratio? What does 4:3 mean? What does 16:9 mean?** *The ratio compares the width to the height. A 4:3 ratio means for every 4 units of width there are 3 units of height; similarly, for every 16 units of width there are 9 units of height.*
- **Which aspect ratio has the greater area for a given diagonal size?** *The 4:3 ratio will yield more area than the 16:9 for a given diagonal length. (This may not be obvious to students without a lot of thought. This particular question is revisited in the EVAULATE.)*
- **What topic from mathematics shows the relationship between the width, height, and diagonal length of a rectangle?** *The Pythagorean Theorem.*

Super Bowl Party

Congratulations! You bragged a little too much about the home theater room that you are adding to your home and your friends have talked you into having a Super Bowl party in a few weeks. The problem is you are missing the all important TV set! You have been shopping for TV sets and have only become confused. You want the largest TV set that will fit the cabinet that you just had custom built for your home theater room. Here is what you have learned.

- TV screens are sized according to the measure of the diagonal of the rectangular screen.
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  - Traditional TV's have an aspect ratio of 4:3 (width: height).
  - High definition TV sets (HDTV) have an aspect ratio of 16:9 (width: height).

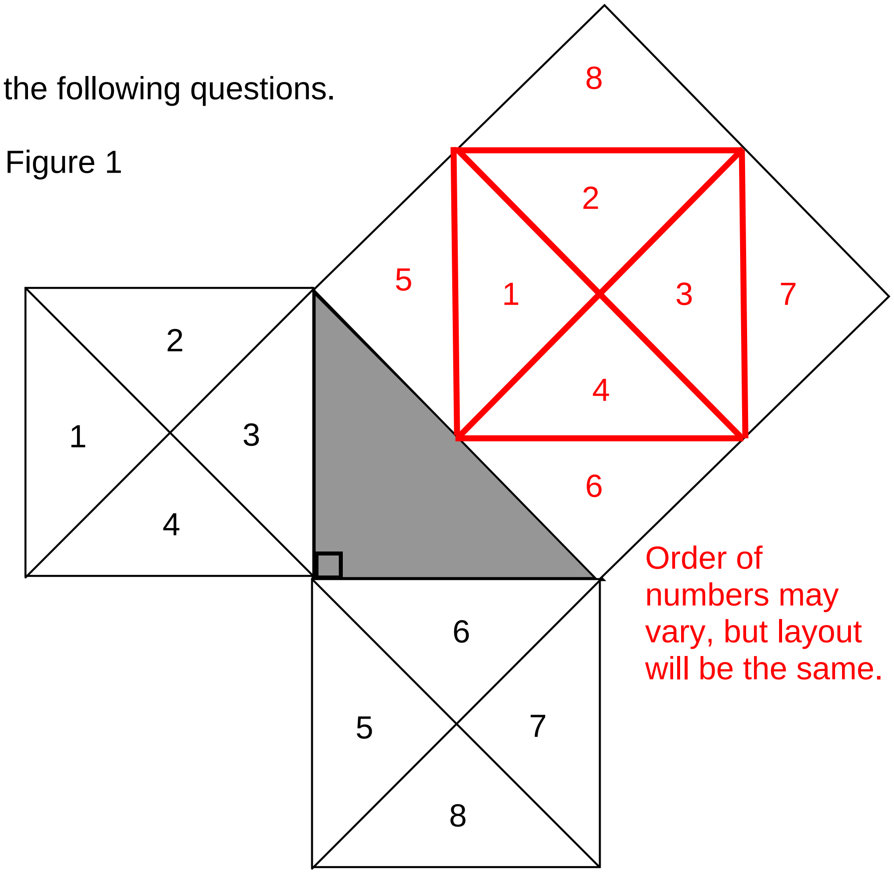
What additional information is needed before you can select the best TV for your home theater room?

Modeling the Pythagorean Theorem (pp. 1 of 6) KEY

In previous courses, you discovered the relationship between the sides of a right triangle. In particular, you discovered that *sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse.* This relationship is TRUE for all right triangles. It is called the Pythagorean Theorem. To understand why the Pythagorean Theorem is true for right triangles, try the following.

Area Model I:
Use Figure 1 for the following questions.

Figure 1



Order of numbers may vary, but layout will be the same.

1. In the figure above, triangles 1-4 form a square with side length that corresponds to one leg of the right triangle. Explain the significance of this part of the model of the Pythagorean Theorem.
   The area of the square represents the square of the leg length.

2. In Figure 1, triangles 5-8 form a square with side length that corresponds to the other leg of the right triangle. Explain the significance of this part of the model of the Pythagorean Theorem.
   The area of the square represents the square of the leg length.

3. In the Figure 1, the large square has a side length that corresponds to the length of the hypotenuse of the right triangle. Explain the significance of this part of the model of the Pythagorean Theorem.
   The area of the large square represents the square of the hypotenuse.

Modeling the Pythagorean Theorem (pp. 2 of 6) KEY

4. How should the area of the squares formed by triangles 1-8 relate to the area of the large square that has a side length that corresponds to the length of the hypotenuse?
   The sum of the areas of the two squares built on the legs of the triangle is equal to the area of the square built on the hypotenuse.

5. Using patty paper, trace the square formed by triangles 1-4. Be sure to trace the triangles also. Color this square using a colored pencil.

6. Using patty paper, trace the square formed by triangles 5-8. Be sure to trace the triangles also. Color this square using a different colored pencil.

7. Cut out triangles 1-8. Arrange the triangles so that they fill in the area bounded by the large square.
   See figure.

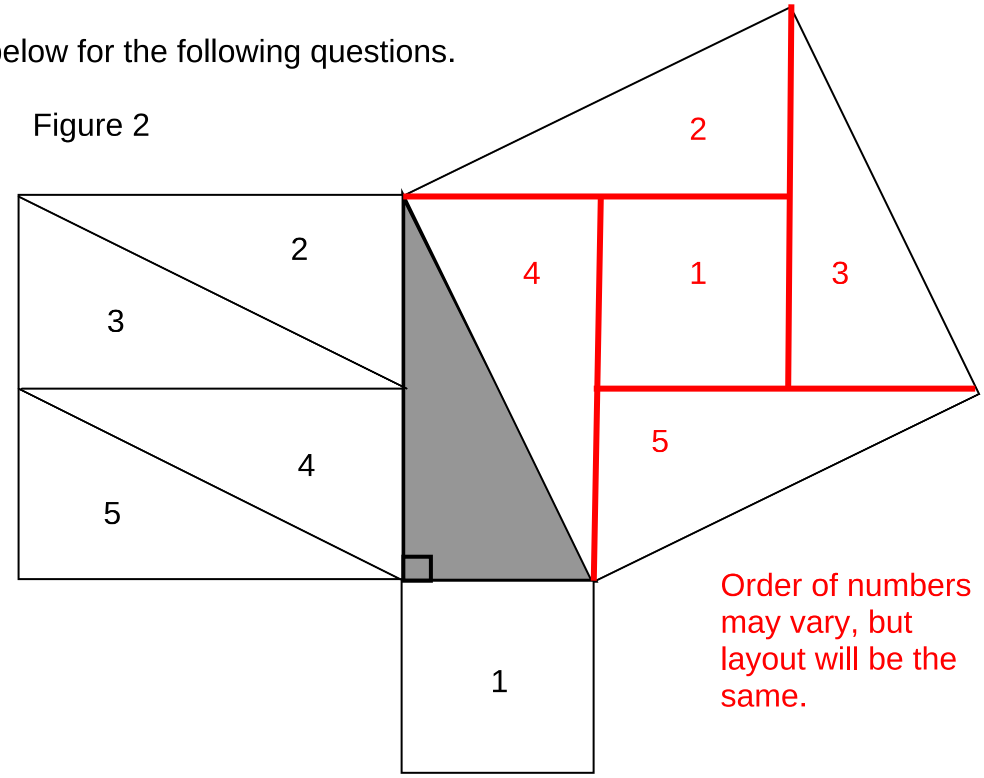
8. Explain how the above models the Pythagorean Theorem.
   The figures that make up the squares of each leg can be rearranged to exactly fill the square of the hypotenuse.

Modeling the Pythagorean Theorem (pp. 3 of 6) KEY

Area Model II:

Use Figure 2 below for the following questions.

Figure 2



Order of numbers may vary, but layout will be the same.

9. In Figure 2, triangles 2-5 form a square with side length that corresponds to one leg of the right triangle. Explain the significance of this part of the model of the Pythagorean Theorem.

   Triangles 2-5 create a square whose area represents the square of the length of one of the legs of the triangle.

10. In the Figure 2, region 1 is a square with side length that corresponds to the short leg of the right triangle. Explain the significance of this part of the model of the Pythagorean Theorem.

    The area of the square region represents the square of the length of the other leg.

11. In figure 2, the large square has a side length that corresponds to the length of the hypotenuse of the right triangle. Explain the significance of this part of the model of the Pythagorean Theorem.

    The area of the square region represents the square of the length of the hypotenuse.

Modeling the Pythagorean Theorem (pp. 4 of 6) KEY

12. Using patty paper, trace the square formed by triangles 2-4. Be sure to trace the triangles also. Color this square using a colored pencil.

13. Using patty paper, trace the small square that has a side length that corresponds to the length of the short leg of the right triangle. Color this square using a different colored pencil.

14. Cut out pieces 1-5 from your tracings. Arrange the pieces so that they fill in the area bounded by the large square.
    See Figure 2.

15. Explain how the above models the Pythagorean Theorem.
    The figures that make up the squares of each leg can be rearranged to exactly fill the square of the hypotenuse.

Modeling the Pythagorean Theorem (pp. 5 of 6) KEY

An Algebraic Approach:
Use the figures below for the following questions.

Figure 1

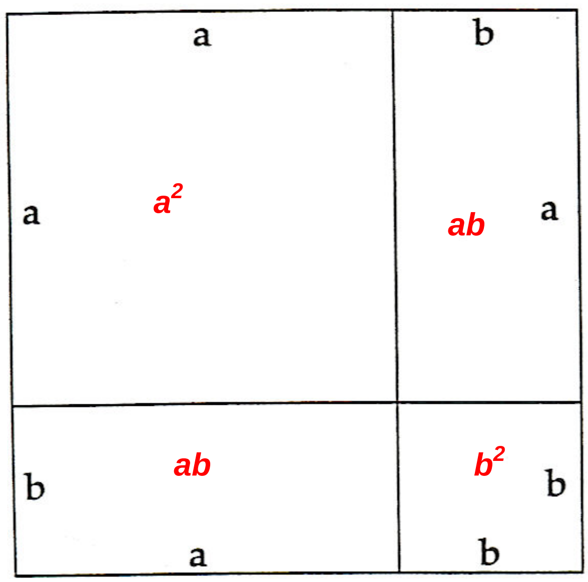
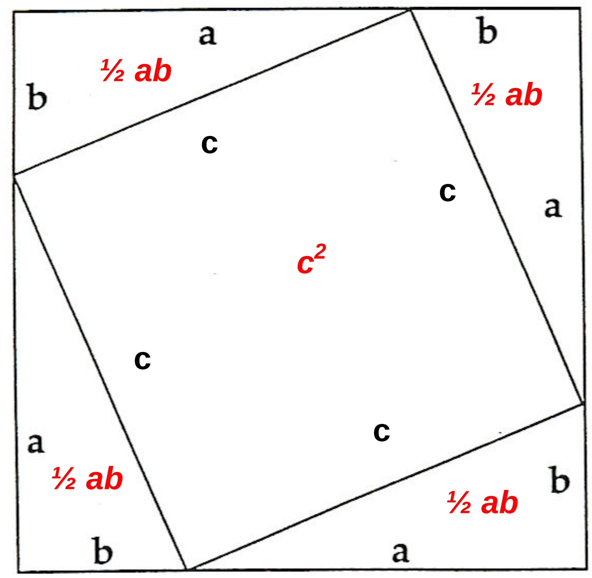


Figure 2



16. Figure 1 and Figure 2 above are both squares. What is the length of each side of Figure 1?
    Figure 1 has length $a + b$.

17. What is the length of each side of Figure 2? What is the scale factor between the two squares?
    Figure 2 has length $a + b$. Scale factor is 1:1.

18. Based on your answer to question 1 and 2, what can you conclude about Figure 1 and Figure 2?
    They are congruent.

19. What is true about the areas of Figure 1 and Figure 2?
    Since the squares are congruent, their areas are equal.

20. Find the areas of each of the pieces that make up Figure 1. Show these areas in the figure above. Write an algebraic expression for the area of Figure 1.
    The area of Figure 1 is $a^2 + 2ab + b^2$.

Modeling the Pythagorean Theorem (pp. 6 of 6) KEY

21. Find the areas of each of the pieces that make up Figure 2. Show these areas in the figure above. Write an algebraic expression for the area of Figure 2.
    The area of Figure 2 is $2ab + c^2$.

22. Based on your answers to 5 and 6, write an equation that states that the area of Figure 1 is equal to the area of Figure 2. Simplify the equation and record your answer below.
    $a^2 + 2ab + b^2 = 2ab + c^2$ simplifies to $a^2 + b^2 = c^2$.

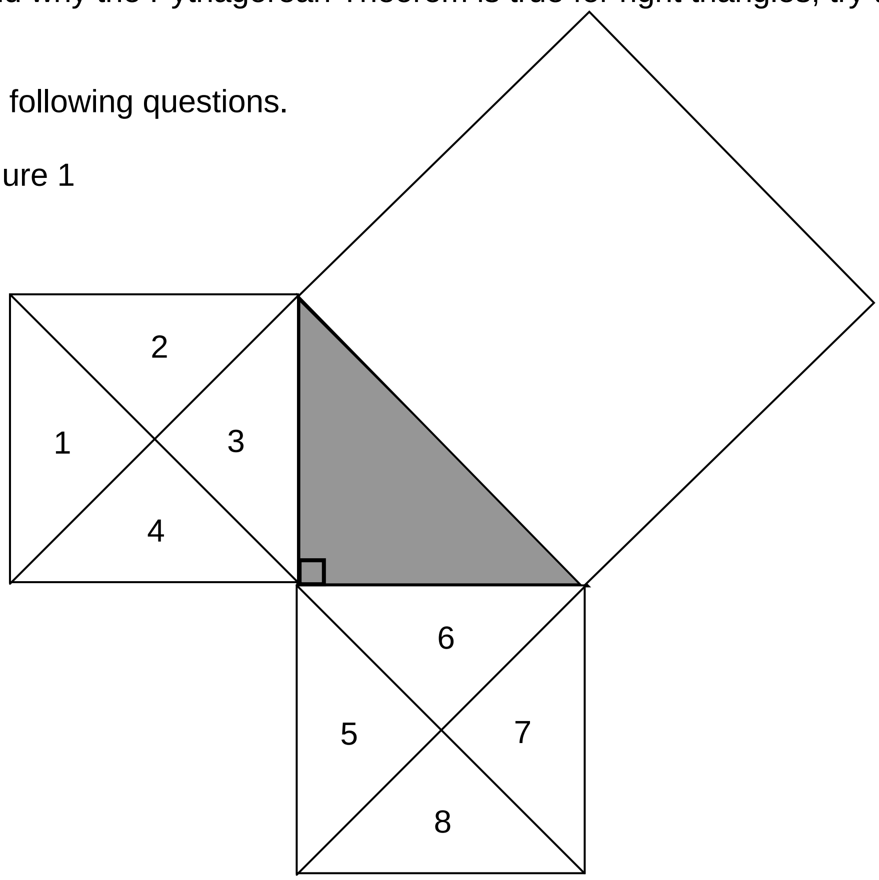
23. What is significant about your equation? How does it relate to Figure 1 and Figure 2?
    Simplifying the equation yields the Pythagorean Theorem. In the figure above, *a* and *b* are the lengths of the legs of the triangles from Figure 2, while *c* is the length of the hypotenuse.

Modeling the Pythagorean Theorem (pp. 1 of 6)

In previous courses, you discovered the relationship between the sides of a right triangle. In particular, you discovered that *sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse.* This relationship that is TRUE for all right triangles is called the Pythagorean Theorem. To understand why the Pythagorean Theorem is true for right triangles, try the following.

Area Model I:
Use Figure 1 for the following questions.

Figure 1



1. In the figure above, triangles 1-4 form a square with side length that corresponds to one leg of the right triangle. Explain the significance of this part of the model of the Pythagorean Theorem.

2. In Figure 1, triangles 5-8 form a square with side length that corresponds to the other leg of the right triangle. Explain the significance of this part of the model of the Pythagorean Theorem.

3. In the Figure 1, the large square has a side length that corresponds to the length of the hypotenuse of the right triangle. Explain the significance of this part of the model of the Pythagorean Theorem.

Modeling the Pythagorean Theorem (pp. 2 of 6)

4. How should the area of the squares formed by triangles 1-8 relate to the area of the large square that has a side length that corresponds to the length of the hypotenuse?

5. Using patty paper, trace the square formed by triangles 1-4. Be sure to trace the triangles also. Color this square using a colored pencil.

6. Using patty paper, trace the square formed by triangles 5-8. Be sure to trace the triangles also. Color this square using a different colored pencil.

7. Cut out triangles 1-8. Arrange the triangles so that they fill in the area bounded by the large square.

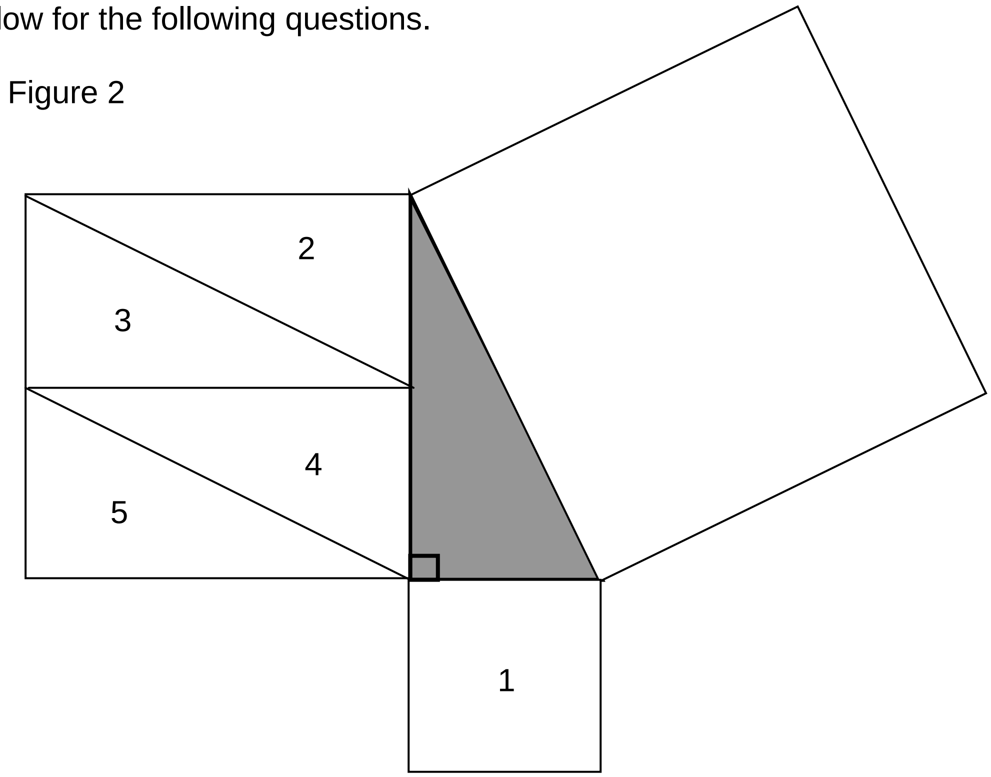
8. Explain how the above models the Pythagorean Theorem.

Modeling the Pythagorean Theorem (pp. 3 of 6)

Area Model II:

Use Figure 2 below for the following questions.

Figure 2



9. In Figure 2, triangles 2-5 form a square with side length that corresponds to one leg of the right triangle. Explain the significance of this part of the model of the Pythagorean Theorem.

10. In the Figure 2, region 1 is a square with side length that corresponds to the short leg of the right triangle. Explain the significance of this part of the model of the Pythagorean Theorem.

11. In Figure 2, the large square has a side length that corresponds to the length of the hypotenuse of the right triangle. Explain the significance of this part of the model of the Pythagorean Theorem.

Modeling the Pythagorean Theorem (pp. 4 of 6)

12. Using patty paper, trace the square formed by triangles 2-4. Be sure to trace the triangles also. Color this square using a colored pencil.

13. Using patty paper, trace the small square that has a side length that corresponds to the length of the short leg of the right triangle. Color this square using a different colored pencil.

14. Cut out pieces 1-5 from your tracings. Arrange the pieces so that they fill in the area bounded by the large square.

15. Explain how the above models the Pythagorean Theorem.

Modeling the Pythagorean Theorem (pp. 5 of 6)

An Algebraic Approach:
Use the figures below for the following questions.

Figure 1

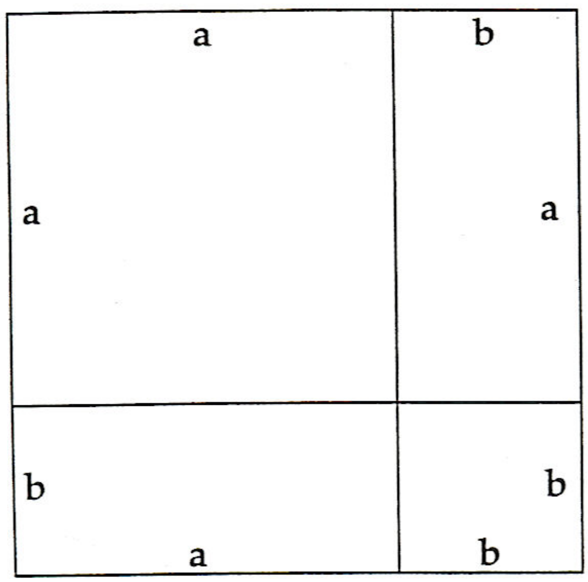
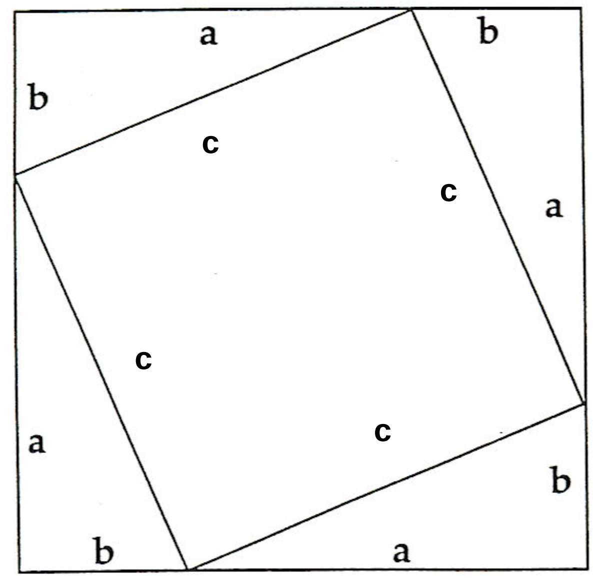


Figure 2



16. Figure 1 and Figure 2 above are both squares. What is the length of each side of Figure 1?

17. What is the length of each side of Figure 2? What is the scale factor between the two squares?

18. Based on your answer to question 1 and 2, what can you conclude about Figure 1 and Figure 2?

19. What is true about the areas of Figure 1 and Figure 2?

20. Find the areas of each of the pieces that make up Figure 1. Show these areas in the figure above. Write an algebraic expression for the area of Figure 1.

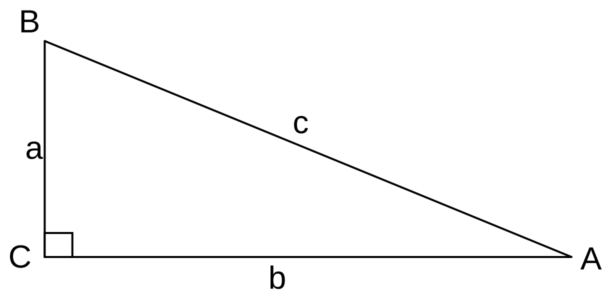
Modeling the Pythagorean Theorem (pp. 6 of 6)

21. Find the areas of each of the pieces that make up Figure 2. Show these areas in the figure above. Write an algebraic expression for the area of Figure 2.

22. Based on your answers to 5 and 6, write an equation that states that the area of Figure 1 is equal to the area of Figure 2. Simplify the equation and record your answer below.

23. What is significant about your equation? How does it relate to Figure 1 and Figure 2?

Pythagorean Theorem Practice (pp. 1 of 3) KEY



Pythagorean Theorem

- If a triangle is a right triangle, then $a^2 + b^2 = c^2$ where *a* and *b* are the lengths of the legs and *c* is the hypotenuse.

Converse of the Pythagorean Theorem

- If *c* is the longest side of a triangle, *a* and *b* are the lengths of the other sides, and $a^2 + b^2 = c^2$, then the triangle is a right triangle.

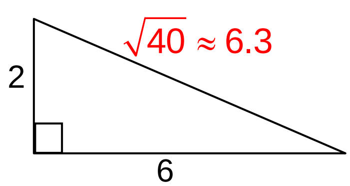
Corollaries to the Converse of the Pythagorean Theorem

- If *c* is the longest side of a triangle, *a* and *b* are the lengths of the other sides, and $a^2 + b^2 < c^2$, then the triangle is an obtuse triangle.
- If *c* is the longest side of a triangle, *a* and *b* are the lengths of the other sides, and $a^2 + b^2 > c^2$, then the triangle is an acute triangle.

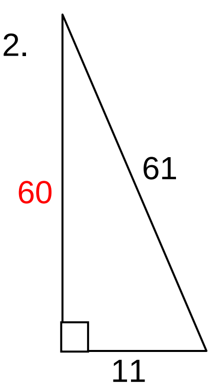
Practice Problems

Use the Pythagorean Theorem to solve for the missing side of each right triangle.

1.



2.



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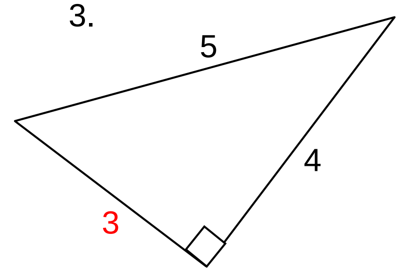

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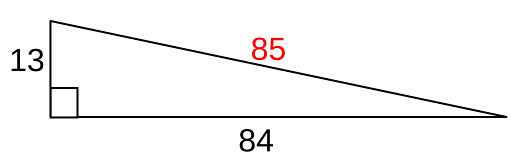

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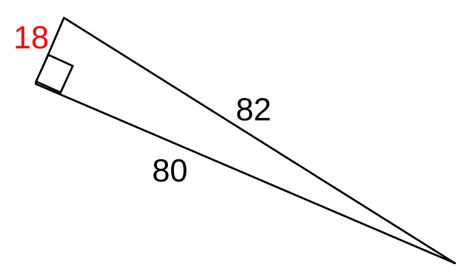
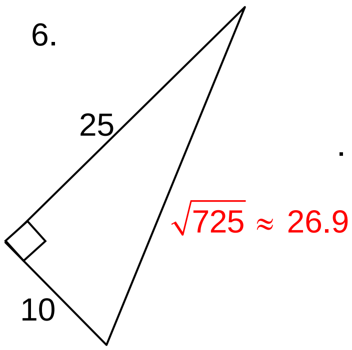


Pythagorean Theorem Practice (pp. 2 of 3) KEY

6.



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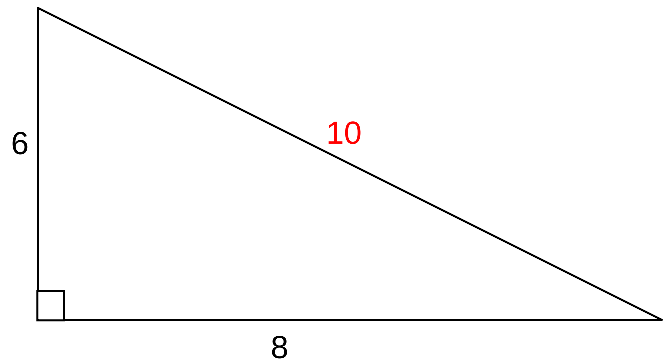

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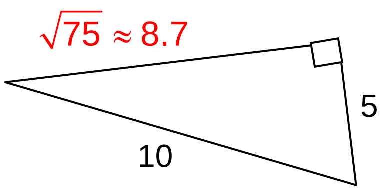
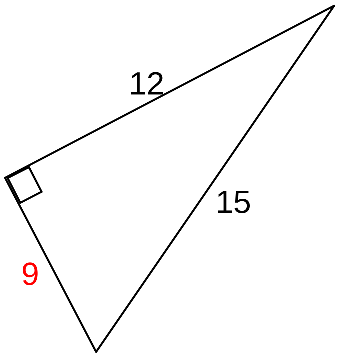
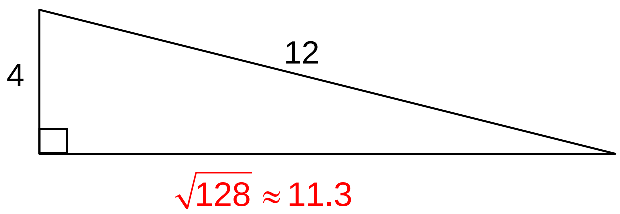


9.



10.



Pythagorean Theorem Practice (pp. 3 of 3) KEY

Use the Converse of the Pythagorean Theorem and its related corollaries to classify the following triangles given their side lengths.

11. 6, 6, 7
Acute

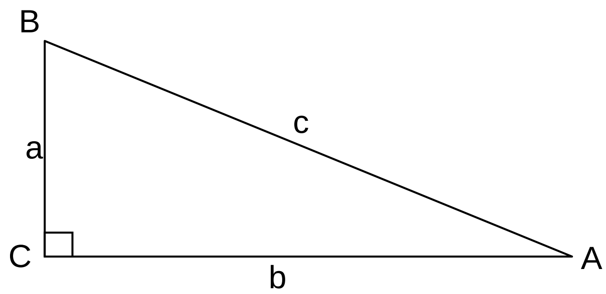
12. 3, 5, 7
Obtuse

13. 4, 4, 4
Acute (Equiangular)

14. 2, 4, 5
Obtuse

15. 7, 24, 25
Right

Pythagorean Theorem Practice (pp. 1 of 3)



Pythagorean Theorem

- If a triangle is a right triangle, then $a^2 + b^2 = c^2$ where *a* and *b* are the lengths of the legs and *c* is the hypotenuse.

Converse of the Pythagorean Theorem

- If *c* is the longest side of a triangle, *a* and *b* are the lengths of the other sides, and $a^2 + b^2 = c^2$, then the triangle is a right triangle.

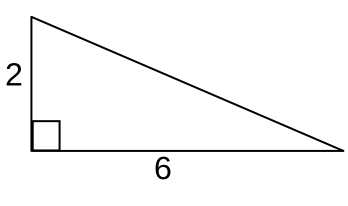
Corollaries to the Converse of the Pythagorean Theorem

- If *c* is the longest side of a triangle, *a* and *b* are the lengths of the other sides, and $a^2 + b^2 < c^2$, then the triangle is an obtuse triangle.
- If *c* is the longest side of a triangle, *a* and *b* are the lengths of the other sides, and $a^2 + b^2 > c^2$, then the triangle is an acute triangle.

Practice Problems

Use the Pythagorean Theorem to solve for the missing side of each right triangle.

1.



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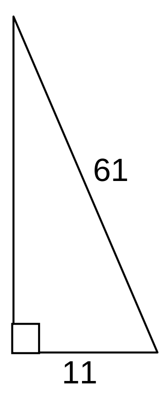

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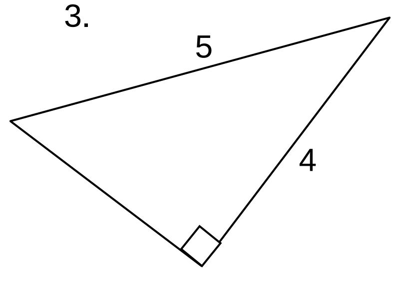

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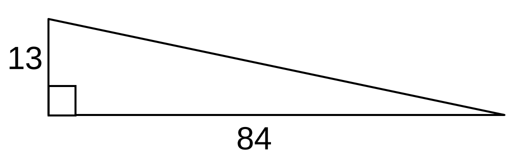
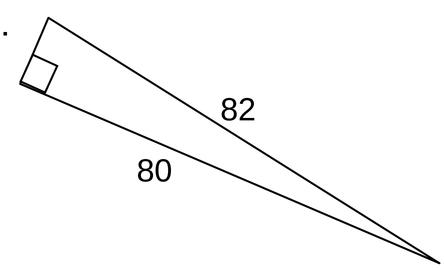
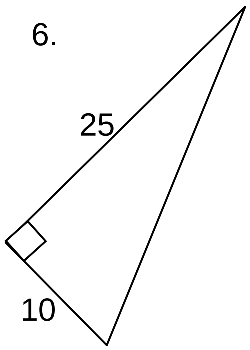


5.



Pythagorean Theorem Practice (pp. 2 of 3)

6.



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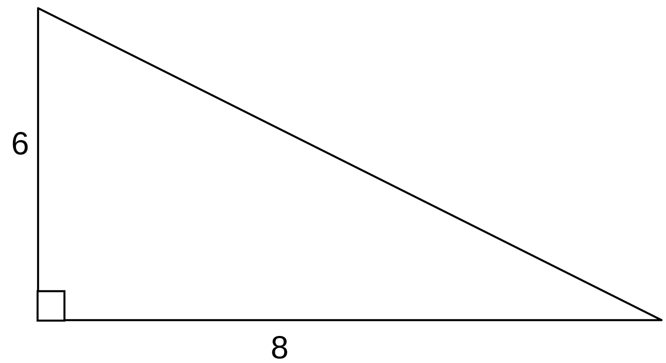

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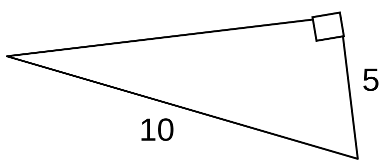

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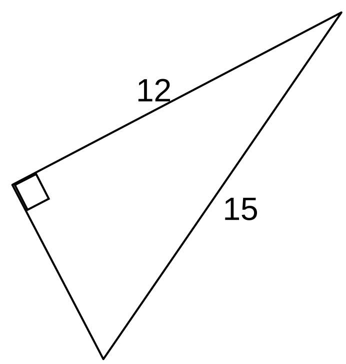
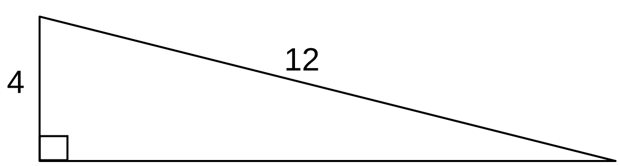


10.



Pythagorean Theorem Practice (pp. 3 of 3)

Use the Converse of the Pythagorean Theorem and its related corollaries to classify the following triangles given their side lengths.

11. 6, 6, 7

12. 3, 5, 7

13. 4, 4, 4

14. 2, 4, 5

15. 7, 24, 25

Discovering Pythagorean Triples (pp. 1 of 3) KEY

In the following exploration activity, you will discover common Pythagorean triples. Pythagorean triples consist of three positive integers, *a*, *b,* and *c*, such that $a^2 + b^2 = c^2$.

1. Using a piece of grid paper and a straight edge, draw a line along one of the vertical lines on your paper. Pick a starting point at one end of your line and label the first grid mark 0. Continue labeling grid marks along the line sequentially. Crease your paper along the vertical line that you drew. You have just created a ruler by which to measure the hypotenuses of triangles you will construct.

2. Using another piece of grid paper, construct the legs of a right triangle using integer lengths. For example, let one leg be 3 units and the other leg 4 units. Use your quad paper ruler from part 1 above to verify that the hypotenuse completes the Pythagorean triple. How long is the hypotenuse?
   5 units

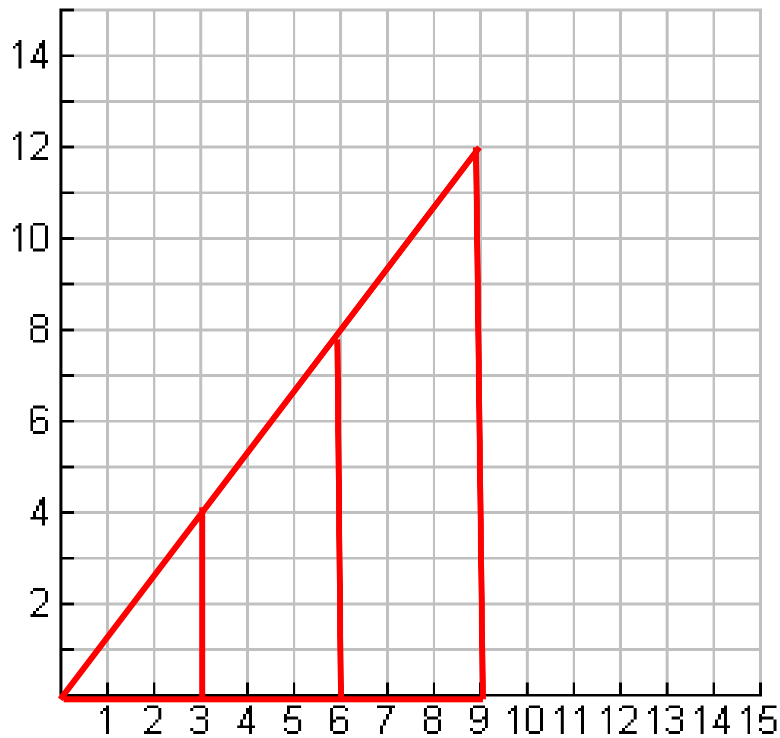
3. Continue creating triangles until you have discovered five other Pythagorean triples. Use the table below to record your findings. Verify numerically that the lengths satisfy the Pythagorean Theorem by completing the columns in the table.
   Student discoveries will vary. Samples given.

Leg	Other Leg	Hypotenuse	Leg Squared	Other Leg Squared	Sum of the Squares of the Legs	Square of the Hypotenuse
a	**b**	**c**	a^2	b^2	$a^2 + b^2$	c^2
3	4	5	9	16	25	25
5	12	13	25	144	169	169
7	24	25	49	576	625	625
8	15	17	64	225	289	289
6	8	10	36	64	100	100
9	12	15	81	144	225	225

4. Do you see any relationships between any two sets of Pythagorean triples that you discovered and recorded in the table above? Explain.
   Students answers will vary according to the data they discover.
   Sample response: The 6, 8, 10 triple is a multiple of the 3, 4, 5 triple.

Discovering Pythagorean Triples (pp. 2 of 3) KEY

Let's take a closer look at Pythagorean triples. Use the graph below for the following questions.



5. Plot the following ordered pairs to form ΔABC. A(0, 0), B(3, 0), and C(3, 4). Dilate ΔABC by a scale factor of 2 and then by a scale factor of 3 using the origin as the center of dilation. Record the vertices of ΔA'B'C' and ΔA''B''C'' in the table below.

Vertices of ΔABC	**Vertices of ΔA'B'C' (Dilation ofΔABC by Scale Factor of 2)**	**Vertices of ΔA''B''C'' (Dilation ofΔABC by Scale Factor of 3)**
A(0, 0)	A'(0, 0)	A"(0, 0)
B(3, 0)	B'(6, 0)	B"(9, 0)
C(3, 4)	C'(6, 8)	C"(9, 12)

6. What are the side lengths of ΔABC? Record your answers in the table on the next page.
   See table.

Discovering Pythagorean Triples (pp. 3 of 3) KEY

7. Use your knowledge of dilations and scale factor to find the side lengths of ΔA'B'C' and ΔA''B''C''. Record your answers in the table below.

Side lengths of ΔABC			Side Lengths of ΔA'B'C'			Side Lengths of ΔA"B"C"		
AB	BC	AC	A'B'	B'C'	A'C'	A"B"	B"C"	A"C"
3	4	5	6	8	10	9	12	15

8. Form the ratios of corresponding sides for ΔABC to ΔA'B'C', and ΔABC to ΔA''B''C''. Record your answers below. What does this lead you to believe about the triangles?
For ΔABC to ΔA'B'C', the common ratio is 1:2.
For ΔABC to ΔA''B''C'', the common ratio is 1:3.
The triangles are similar to each other.

9. How would you represent the relationship between the triangles symbolically?
ΔABC ~ ΔA'B'C' ~ ΔA''B''C''

10. Use the Converse of the Pythagorean Theorem to verify that the side lengths of ΔA'B'C' and ΔA''B''C'' form right triangles. Show your work below.

For ΔA'B'C': $6^2 + 8^2$? 10^2
36 + 64 ? 100
100 = 100
Right Triangle

For ΔA''B''C'': $9^2 + 12^2$? 15^2
81 + 144 ? 225
225 = 225
Right Triangle.

11. Given a Pythagorean triple, write a conjecture about how to generate more Pythagorean triples based on your discovery above.
Multiplying a Pythagorean triple by a positive integer results in another Pythagorean triple.

Discovering Pythagorean Triples (pp. 1 of 3)

In the following exploration, you will discover common Pythagorean triples. Pythagorean triples consist of three positive integers, *a*, *b* and *c*, such that $a^2 + b^2 = c^2$.

1. Using a piece of quad paper and a straight edge, draw a line along one of the vertical lines on your paper. Pick a starting point at one end of your line and label the first grid mark 0. Continue labeling grid marks along the line sequentially. Crease your paper along the vertical line that you drew. You have just created a ruler by which to measure the hypotenuses of triangles you will construct.

2. Using another piece of quad paper, construct the legs of a right triangle using integer lengths. For example, let one leg be 3 units and the other leg 4 units. Use your quad paper ruler from part 1 above to verify that the hypotenuse completes the Pythagorean triple. How long is the hypotenuse?

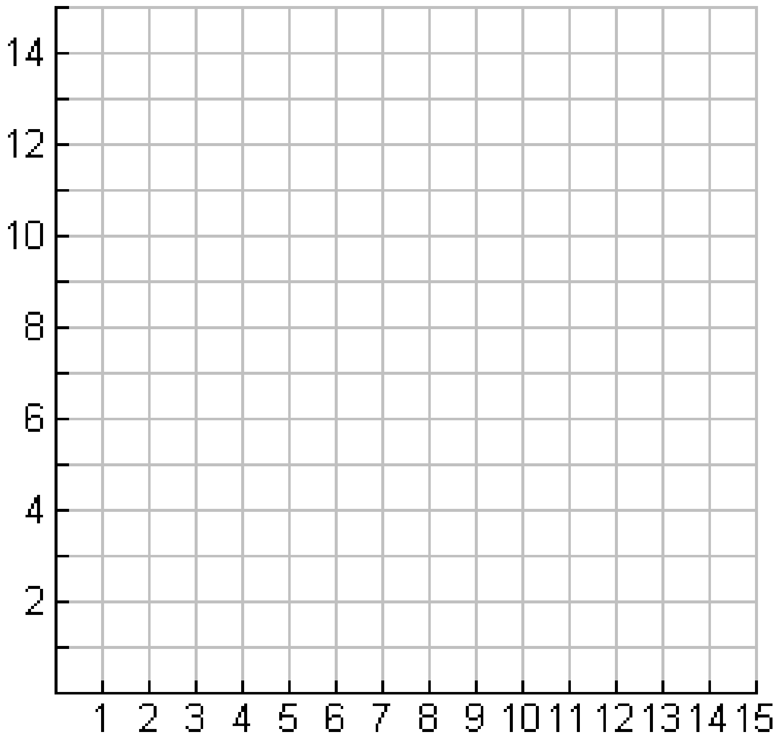
3. Continue creating triangles until you have discovered five other Pythagorean triples. Use the table below to record your findings. Verify numerically that the lengths satisfy the Pythagorean Theorem by completing the columns in the table.

Leg	Other Leg	Hypotenuse	Leg Squared	Other Leg Squared	Sum of the Squares of the Legs	Square of the Hypotenuse
a	**b**	**c**	a^2	b^2	$a^2 + b^2$	c^2
3	4	5	9	16	25	25
5	12					
7		25				
	15	17				

4. Do you see any relationships between any two sets of Pythagorean triples that you discovered and recorded in the table above? Explain.

Discovering Pythagorean Triples (pp. 2 of 3)

Let’s take a closer look at Pythagorean triples. Use the graph below for the following questions.



5. Plot the following ordered pairs to form ΔABC. A(0, 0), B(3, 0), and C(3, 4). Dilate ΔABC by a scale factor of 2 and then by a scale factor of 3 using the origin as the center of dilation. Record the vertices of ΔA’B’C’ and ΔA’’B’’C’’ in the table below.

Vertices of ΔABC	**Vertices of ΔA’B’C’ (Dilation of ΔABC by Scale Factor of 2)**	**Vertices of ΔA’’B’’C’’ (Dilation of ΔABC by Scale Factor of 3)**
A(0, 0)		
B(3, 0)		
C(3, 4)		

6. What are the side lengths of ΔABC? Record your answers in the table on the next page.

Discovering Pythagorean Triples (pp. 3 of 3)

7. Use your knowledge of dilations and scale factor to find the side lengths of ∆A'B'C' and ∆A''B''C''. Record your answers in the table below.

Side lengths of ∆ABC			Side Lengths of ∆A'B'C'			Side Lengths of ∆A"B"C"		
AB	**BC**	**AC**	**A'B'**	**B'C'**	**A'C'**	**A"B"**	**B"C"**	**A"C"**

8. Form the ratios of corresponding sides for ∆ABC to ∆A'B'C', and ∆ABC to ∆A''B''C''. Record your answers below. What does this lead you to believe about the triangles?

9. How would you represent the relationship between the triangles symbolically?

10. Use the Converse of the Pythagorean Theorem to verify that the side lengths of ∆A'B'C' and ∆A''B''C'' form right triangles. Show your work below.

11. Given a Pythagorean triple, write a conjecture about how to generate more Pythagorean triples based on your discovery above.

Applications of Pythagorean Theorem and Pythagorean Triples (pp. 1 of 2) KEY

In the previous exploration, you learned that Pythagorean triples consist of three positive integers, *a*, *b,* and *c*, such that $a^2 + b^2 = c^2$, and that Pythagorean triples can be generated by multiplying a given Pythagorean triple by any positive integer.

Some common Pythagorean triples are listed in the table below. Scale each triple by a scale factor of 2 and then by 3. Use the Pythagorean Theorem or the information in the table to help solve the problems that follow.

Pythagorean Triple	Scale Factor of 2	Scale Factor of 3
3, 4, 5	6, 8, 10	9, 12, 15
5, 12, 13	10, 24, 26	15, 36, 39
7, 24, 25	14, 48, 50	21, 72, 75
8, 15, 17	16, 30, 34	24, 45, 51

Draw a pictorial representation of each problem and solve.

1. The length of one side of a rectangle is 4. The length of the diagonal is 5. Find the length of the other side.
   3 units

2. The lengths of the two congruent sides of an isosceles triangle are 5 m. The base of the triangle is 6 m. Find the length of the altitude.
   4 m

3. What is the altitude of an equilateral triangle with side length of 6 yards? Find its area.
   $\text{Altitude}=\sqrt{27} \approx 5.2$ yards
   $\text{Area}=9\sqrt{3} \approx 15.6$ sq. yds.

4. A rectangle is 5 meters wide and 12 meters long. How long is the diagonal of the rectangle?
   13 m

5. A rectangle is 18 centimeters long with a diagonal of 22.2 cm. How wide is the rectangle?
   ≈ 13.0 cm

Applications of Pythagorean Theorem and Pythagorean Triples (pp. 2 of 2) KEY

6. A guy wire is attached to an upright pole 6 meters above the ground. If the wire is anchored to the ground 4 meters from the base of the pole, how long is the wire?
   $\sqrt{52} \approx 7.2\text{ m}$

7. The size of a television is given by the diagonal measure of its screen. A 37.2-inch TV has a width of 30 in. How tall is the television? What is the area of the screen?
   Height $\approx$ 22.0 in
   Area $\approx$ 660 sq. in.

8. A ship leaves port and sails 14 kilometers west and then 48 kilometers north. How far is the ship from port?
   50 km

9. Each side of a square checkerboard measures 40 cm. What is the length of the diagonal of the board?
   $\sqrt{3200} \approx 56.6\text{ cm}$

10. A 9.8 meter inclined ramp rises over a horizontal distance of 9 meters. How tall is the ramp at its highest point?
    3.9 m

11. A television has a width: height ratio of 9:5. If the television is advertised as having a 36 inch screen (remember the screen is measured as the diagonal), what is the actual height and width of the television? What is the surface area of the screen?
    $\sqrt{((9x)^2 + (5x)^2)} = 36$
    $\sqrt{(81x^2 + 25x^2)} = 36$
    $106x^2 = 1296$
    $x = 3.5$
    Therefore the width is 9(3.5) = 31.5 inches, and the height is 5(3.5) = 17.5 inches.
    The area is (31.5 x 17.5) = 554.75 square inches.

12. The window of a burning building is 24 meters above the ground. The base of a ladder is placed 10 meters from the building. How long must the ladder be to reach the window?
    26 m

Applications of Pythagorean Theorem and Pythagorean Triples (pp. 1 of 2)

In the previous exploration, you learned that Pythagorean triples consist of three positive integers, *a*, *b* and *c*, such that $a^2 + b^2 = c^2$, and that Pythagorean triples can be generated by multiplying a given Pythagorean triple by any positive integer.

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Pythagorean Triple	Scale Factor of 2	Scale Factor of 3
3, 4, 5		
5, 12, 13		
7, 24, 25		
8, 15, 17		

Draw a pictorial representation of each problem and solve.

1. The length of one side of a rectangle is 4. The length of the diagonal is 5. Find the length of the other side.

2. The lengths of the two congruent sides of an isosceles triangle are 5 m. The base of the triangle is 6 m. Find the length of the altitude.

3. What is the altitude of an equilateral triangle with side length of 6 yards? Find its area.

4. A rectangle is 5 meters wide and 12 meters long. How long is the diagonal of the rectangle?

5. A rectangle is 18 centimeters long with a diagonal of 22.2 cm. How wide is the rectangle?

Applications of Pythagorean Theorem and Pythagorean Triples (pp. 2 of 2)

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12. The window of a burning building is 24 meters above the ground. The base of a ladder is placed 10 meters from the building. How long must the ladder be to reach the window?

Super Bowl Party Revisited KEY

The Super Bowl is just weeks away and you still have not bought a TV. You have decided that you really like the 42" screens but you still can not decide between the traditional TV which has a 4:3 width: height ratio, or the newer HDTV which has a 16:9 width: height ratio. The cabinet that you had built to house the TV has an opening that is exactly 42" wide! At first, you think you will have to down size your TV to make it fit, but then you remember that TV screens are sized according to the diagonal length. You decide to determine if either 42" format will fit the cabinet.

1. Analysis of the 4:3 ratio traditional TV
   a. Since the width: height ratio of the screen is 4:3, what number would represent the diagonal of the screen? Write a ratio of height: width: diagonal.
      5 Ratio of height: width: diagonal is 3:4:5.
   b. What is the significance of the ratio you wrote in part *a*?
      It is a Pythagorean triple.
   c. Use the ratio you wrote in part *b* to determine the width and height of the TV screen.
      Width: 33.6 in, height: 25.2 in
   d. What is the area in square inches of the TV screen?
      846.72 in^2
   e. Allowing 2" of clearance at either end of the 42" cabinet opening leaves you with 38" of usable opening. Will the 4:3 ratio TV set fit the opening?
      Yes.

2. Analysis of the 16:9 ratio HDTV
   a. What is the symbolic representation of the width and height of the rectangle in a 16:9 ratio in terms of *x*?
      W = 16x H = 9x
   b. Find the value of *x* and determine the width and height of the 42" TV screen.
      $x \approx 2.3$, Width: 36.6 in, Height: 20.6 in
   c. What is the area in square inches of the TV screen?
      753.96 in^2
   d. Allowing 2" of clearance at either end of the 42" cabinet opening leaves you with 38" of usable opening. Will the 4:3 ratio TV set fit the opening?
      Yes.

3. Which of the TV screen formats will fit the cabinet opening? Which format offers the most viewing area in square inches?
   Either will fit the opening. The 4:3 format offers the most square inches of viewing area.

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